ON THE PRACTICALITY OF SUPERDIRECTIVITY IN THE HF BAND

Songyi Yen, Edward R. Wawrzynek, and Dejan S. Filipovic
Department of Electrical, Computer, and Energy Engineering
University of Colorado, Boulder, CO, USA, 80309

Superdirectivity has been and remains an exciting concept to maximize the radiation of a given array in the desired direction. Many attempts are scattered throughout the open literature by engineers and scientists hoping to cage this elusive fairy. Herein, we re-visit recent work examining the practicality of this concept for the HF band, where efficiencies on receive are much less a concern due to the dominance of external noise. Analytical results in the ideal case are first discussed followed by considerations on the sensitivity of the excitations. Afterwards, application to simulated arrays of half-wave dipoles as well as electrically small monopoles and folded helical antennas is provided. While directivity of such arrays may be increased using superdirective excitations, many detriments limit their practical usefulness in the HF band. These include sensitivity of the excitations and necessity of compensation using the embedded element patterns. Furthermore, it is found that the superdirective excitations investigated within are best suited for low number of elements. In general, but also as element number increases, the Hansen-Woodyard endfire array remains efficient, robust, and largely agnostic to element type.

1. Introduction

Superdirectivity is a natural follow-on from array synthesis theory to push the limits of what is possible. First discussed in the previous century [1], this concept has attracted many engineers and scientists to realize its claims. Time after time, however, practical superdirectivity has shown itself elusive. This is due to a multitude of factors, including low efficiency, sensitive tolerances, and high localized currents and voltages, to name a few [2]. Some more recent works claim several of these issues can be ignored, or at the very least are mitigated, in the HF band on receive [3], [4]. The authors of these works argue that this is the case due to the high external noise that renders low efficiency of superdirective excitations a non-issue and improvements in technology like software defined radios for the tolerances. The aim of this paper is to show that while these realities may be the case, the glaring difficulties of exacting excitations remain to keep superdirectivity an impracticality even with the advantages found in the HF band. Of course there are ways to address these problems, but to do so in a practical realization remains an open challenge. This is due to requiring more complete knowledge of the system than is typically known (e.g. exact *in situ* embedded element patterns). Accordingly, it is the conclusion of the authors that the Hansen-Woodyard superdirective excitation [5] remains the primary approach for endfire arrays, at least for the time being.

2. Efficiency for Receive at HF

The primary argument for superdirectivity at HF is driven by dominance of external noise rendering efficiency irrelevant on receive. This is expressed in (1), adapted from [3], where η is the efficiency, S_{ext} is the desired signal, $D(\theta, \phi)$ is the directivity in the direction of the incident signal, and N_{ext} and N_{int} are the external and internal noise, respectively.

$$SNR = \left. \frac{\eta S_{ext} D(\theta, \phi)}{\eta N_{ext} + N_{int}} \right|_{\eta N_{ext} \gg N_{int}} = \frac{S_{ext} D(\theta, \phi)}{N_{ext}} \tag{1}$$

That external noise dominates in the HF band is well understood with 10-30 dB greater levels over internal noise for over-the-horizon radar receivers [6] and noise temperatures in excess of 30,000 to even millions of Kelvins in the HF band [7]. All this suggests that efficiency on receive is of lesser importance. This reasoning is similar to why impedance match is often not as important on receive [6]—any enhancement improves the reception of noise by the same amount! Of course, the efficiency cannot be so low that external noise is brought to the level of internal noise.

3. Generating Superdirective Excitations

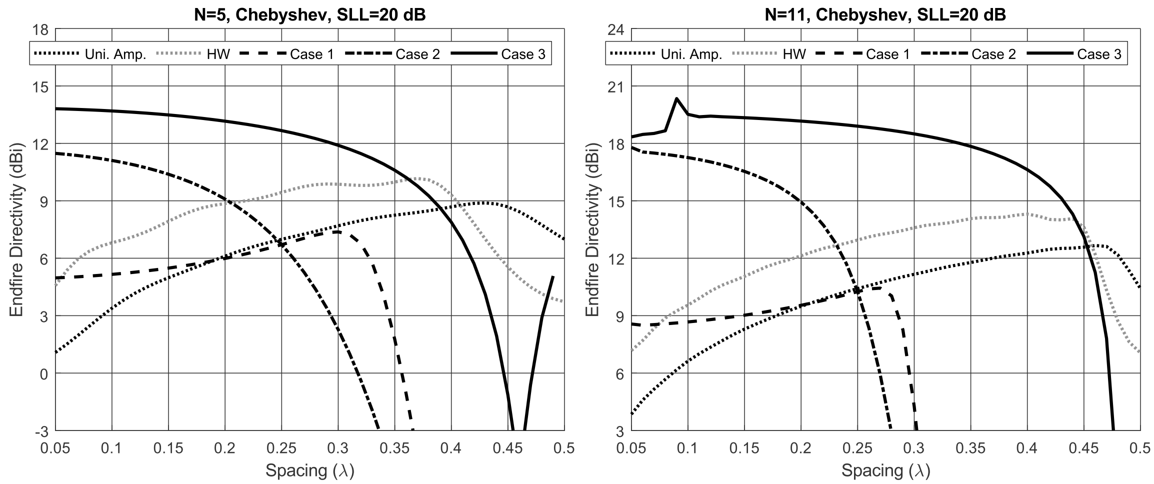
Different methods of generating superdirective excitations have been investigated in the past. For example, early approaches used an optimization approach to maximize directivity, recognizing the quadratic form of the definition of directivity [8]. Herein, we follow [9], [10] in using the Chebyshev polynomial to generate superdirective endfire excitations. There are three cases: 1) "other than broadside case" which is used to steer the pattern away from broadside and towards endfire; 2) "endfire case" which specifically is used to design endfire arrays; and 3) "optimum endfire case" which in essence doubles the visible region by folding it upon itself. The latter case leads to twice as many side lobes as normal, lower beamwidth, and therefore increased directivity. For comparison, uniform amplitude phase steering and the Hansen-Woodyard excitation [5] are included as well.

4. Superdirective Array Factors

First, we examine the array factors. The endfire directivities of array factors with 5 and 11 elements are plotted in Fig. 1. We see that Cases 2 and 3 exhibit superdirectivity up to 0.25λ and 0.4-0.45λ spacing, respectively. This is especially true at smaller element spacings. Conversely, Case 1 is largely invalidated by the Hansen-Woodyard excitation.

Patterns for 5 and 11 elements at 0.15λ and 0.25λ elements spacings are plotted in Fig. 2. It is clear how Case 3 has more side lobes, narrower beamwidth, and therefore higher directivity. We also see how at 0.25λ spacing, Case 2 and uniform amplitude resemble each other. The Hansen-Woodyard excitation directivity remains consistently higher than uniform amplitude, and Case 3 exceeds all the other excitations.

For completeness, the excitation values for 5 elements at 0.15λ spacing (Fig. 2(a)) are given in Table I. The uniform amplitude phase gradient is simply $-kd$, where k is the free-space



(a) (b)

Figure 1. Endfire directivities of linear array factors with (a) 5 and (b) 11 elements. Note the difference in vertical axis.

wave number and d is the element spacing. Notable, the highest directivity excitation of Case 3 is nearly anti-phase between the elements, which is emblematic of the efficiency issues that appear when applying these excitations to elements with realistic losses [2].

Table I. Excitations for 5 elements at 0.15λ spacing.

Uni. Amp.	Hansen-Woodyard	Case 1	Case 2	Case 3
$1\angle 108^\circ$	$1\angle 175.4^\circ$	$0.44\angle 108^\circ$	$0.20\angle -108^\circ$	$0.14\angle -14.9^\circ$
$1\angle 54^\circ$	$1\angle 87.7^\circ$	$0.14\angle -126^\circ$	$0.50\angle 126^\circ$	$0.49\angle 172.5^\circ$
$1\angle 0^\circ$	$1\angle 0^\circ$	$0.76\angle 0^\circ$	$0.65\angle 0^\circ$	$0.69\angle 0^\circ$
$1\angle -54^\circ$	$1\angle -87.7^\circ$	$0.14\angle 126^\circ$	$0.50\angle -126^\circ$	$0.49\angle -172.5^\circ$
$1\angle -108^\circ$	$1\angle -175.4^\circ$	$0.44\angle -108^\circ$	$0.20\angle 108^\circ$	$0.14\angle 14.9^\circ$

5. Sensitivity of Excitations

While the results in the previous section show promise, one of the major issues with superdirective arrays has been and remains the sensitivity/tolerance of the excitations [2]. To illustrate this, the ideal excitations are perturbed by uniformly distributed errors of ±5% of the maximum amplitude and ±5º. A Monte Carlo analysis is run with 1000 runs and histogram results plotted in Fig. 3 for 5 and 11 elements with 0.15λ and 0.25λ spacings. Vertical black line denotes endfire directivity without errors.

For 5 elements, we see that Case 2 and Case 3 outperforms the Hansen-Woodyard excitation for 0.15λ and 0.25λ spacing, respectively. However, when the number of elements is increased to 11, the Hansen-Woodyard excitation is the clear best option as the superdirective excitations either suffer extreme sensitivity or have lower directivity to begin with. These results suggest that these superdirective excitations are only suitable for a low number of elements.

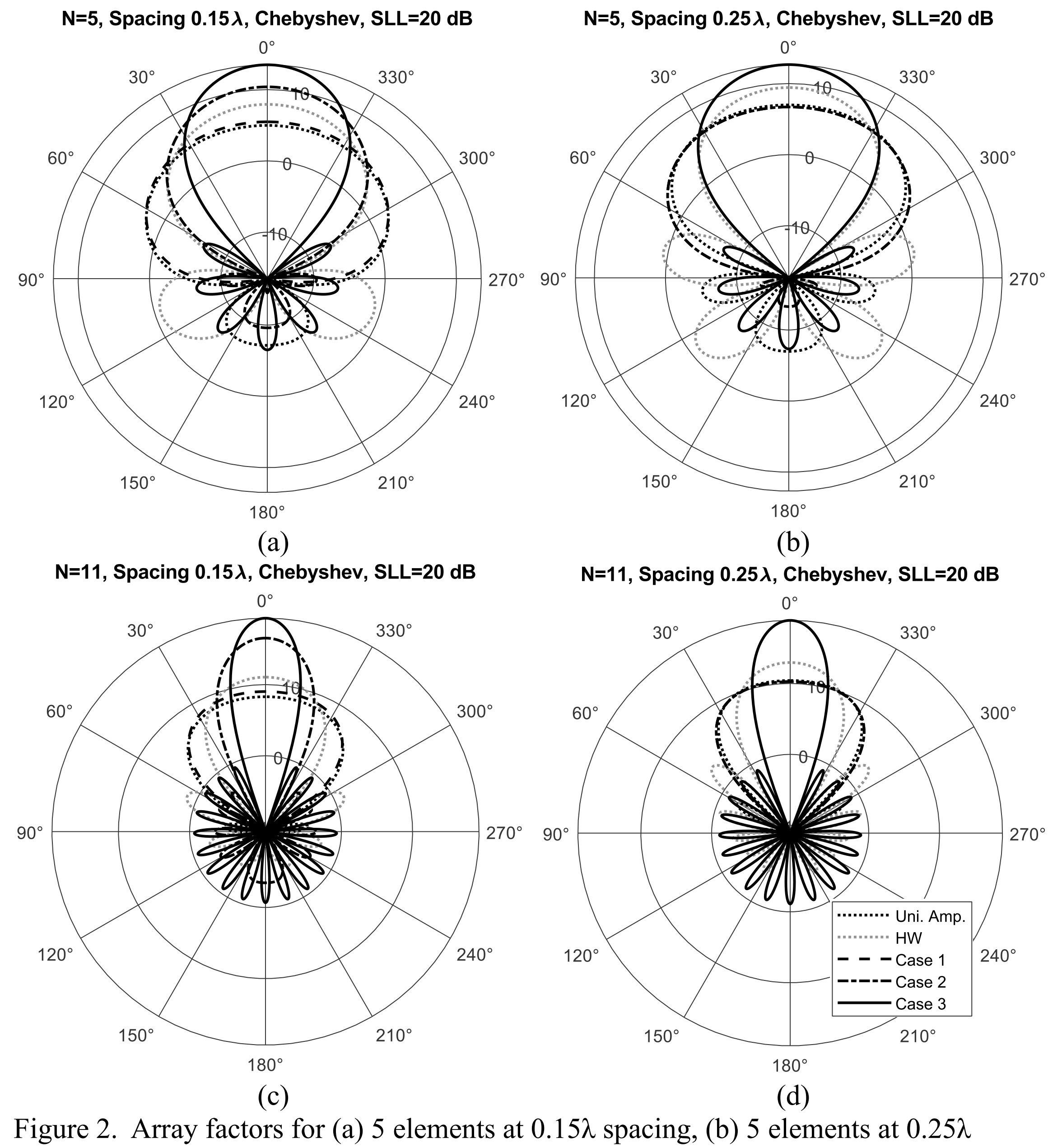


Figure 2. Array factors for (a) 5 elements at 0.15λ spacing, (b) 5 elements at 0.25λ spacing, (c) 11 elements at 0.15λ spacing, and (d) 11 elements at 0.25λ spacing.

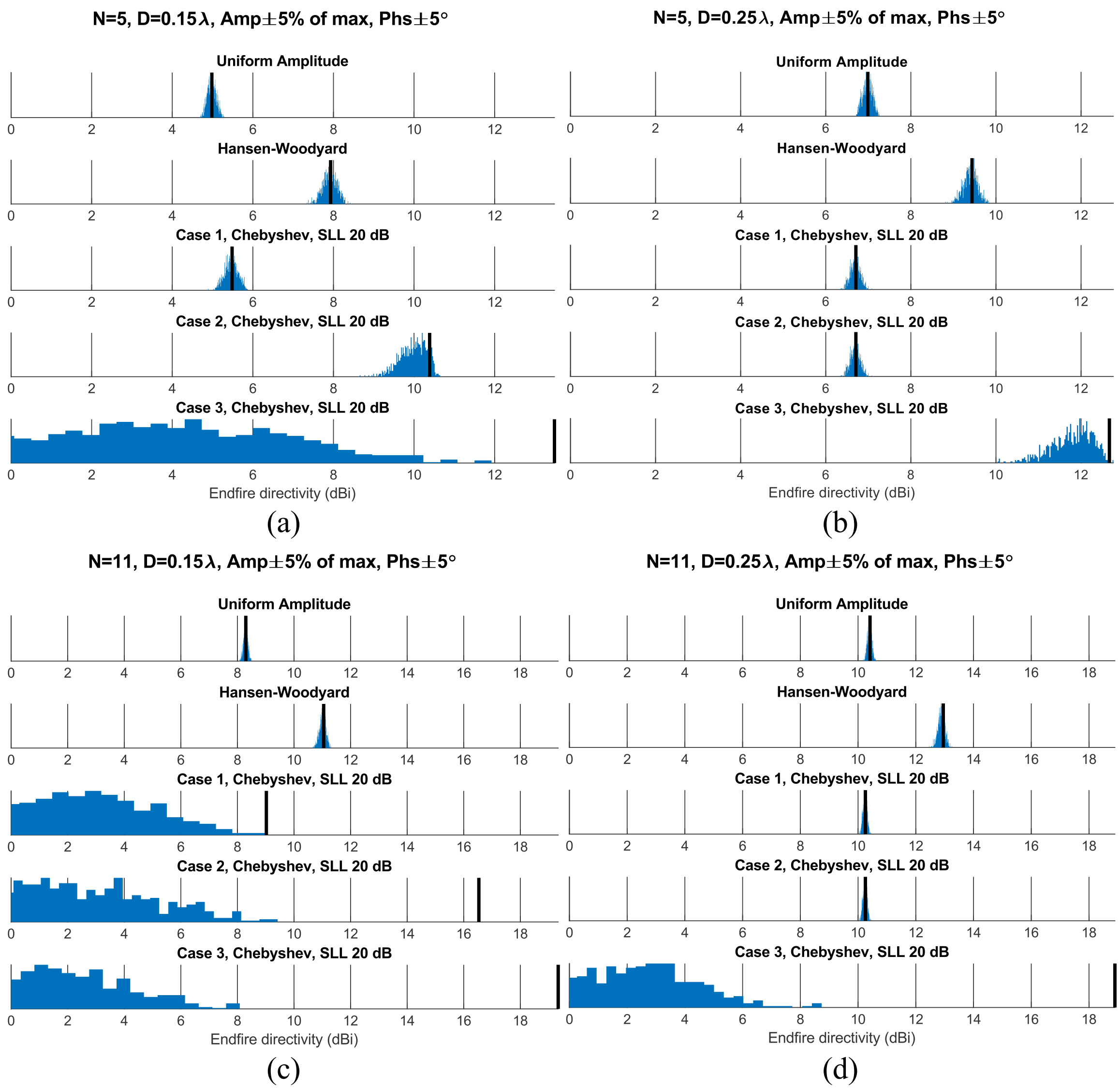


Figure 3. Histograms showing distribution of endfire directivity for Monte Carlo analysis runs. Vertical black line denotes directivity of excitation without errors. (a) 5 elements at 0.15λ spacing, (b) 5 elements at 0.25λ spacing, (c) 11 elements at 0.15λ spacing, and (d) 11 elements at 0.25λ spacing.

Array factors for 5 elements and 0.15λ and 0.25λ spacings are shown in Fig. 4. Immediately apparent is the relative stability of the Hansen-Woodyard excitation array factor compared to Cases 2 and 3. Notably, the sidelobes and backlobes of the Case 2 and 3 excitations show dramatic variation across the Monte Carlo runs.

6. Application to Dipole Array

5 element arrays of copper half-wave dipoles (0.478λ length) are simulated in Altair Feko varying the spacing. The directivity of an array with 0.15λ spacing and the desired array

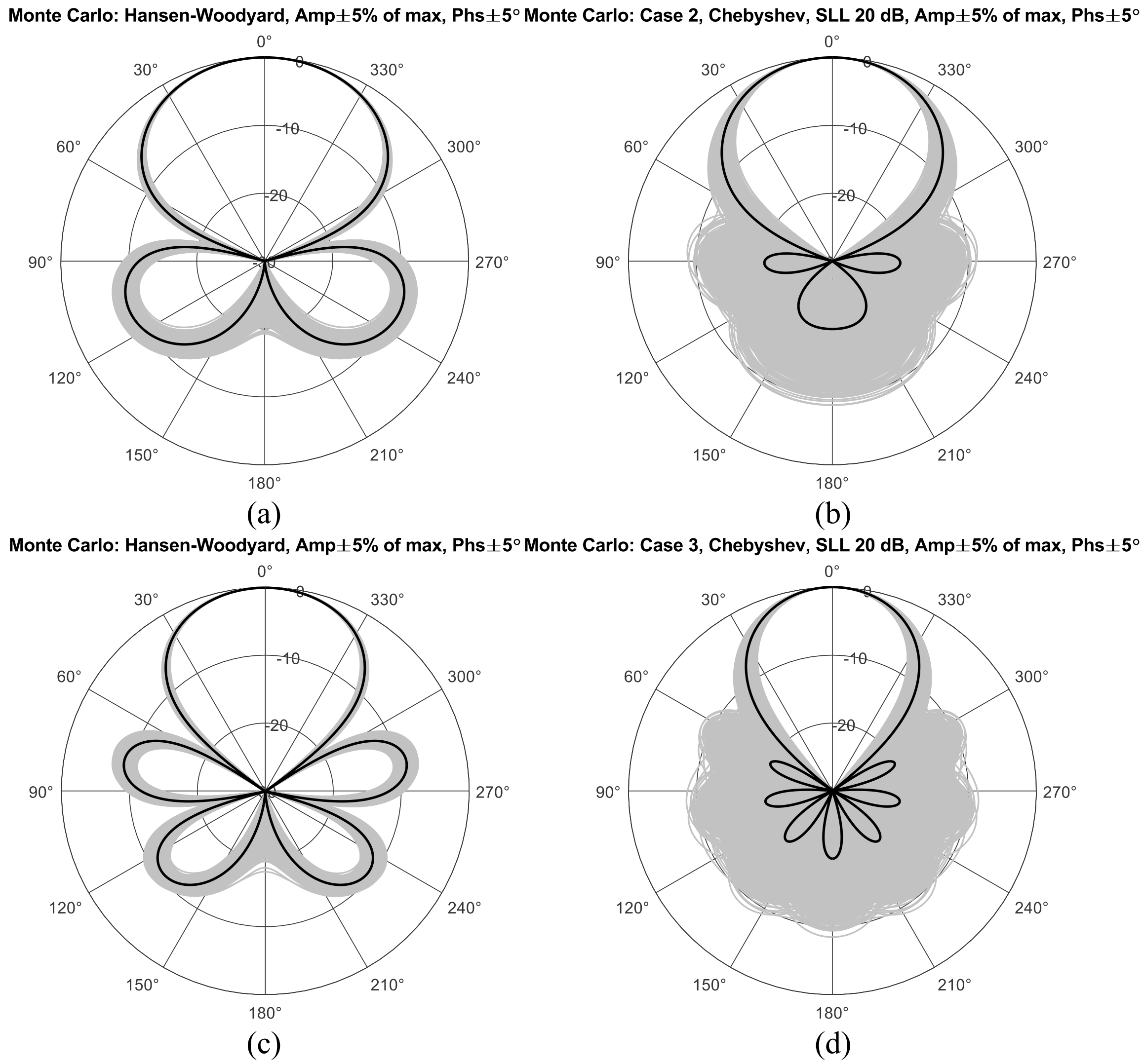
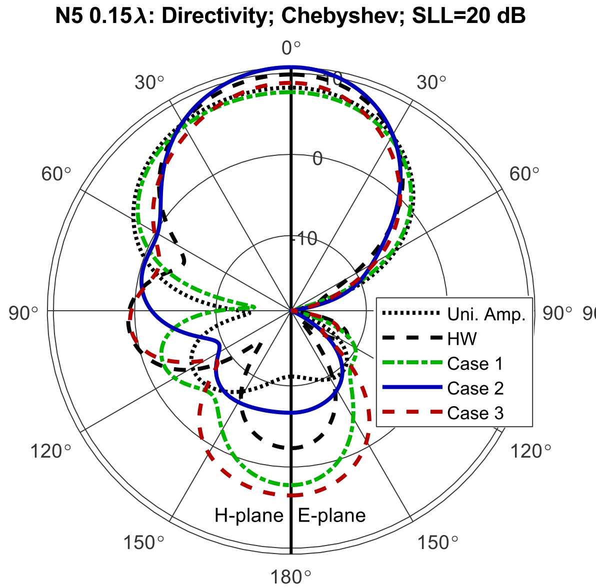


Figure 4. Array factors resulting from Monte Carlo runs. 5 element arrays. Black array factor corresponds to excitation without errors. (a) Hansen-Woodyard excitation on 0.15λ spacing. (b) Case 2 excitation on 0.15λ spacing. (c) Hansen-Woodyard excitation on 0.25λ spacing. (d) Case 3 excitation on 0.25λ spacing.

factor are plotted in Fig. 5. Immediately apparent, directly applying the superdirective excitations to the simulated array leads to pattern distortions compared to the desired array factor. However, the uniform amplitude and Hansen-Woodyard excitations exhibit reasonable agreement to the array factor and showing the increase in directivity due to the element directivity as well.

The poor performance of the superdirective excitations is due to the non-isotropic embedded element patterns (EEPs). In other words, the EEPs impose a non-uniform weighting on the excitations when transferring to the far-field. For the superdirective excitations, owing to their sensitivity as seen in the previous section, this causes significant



(a) (b)

Figure 5. 5 element half-wave dipole array simulated in Altair Feko. 0.15λ spacing. (a) Directivity of endfire array. (b) Desired array factor.

pattern distortions. This can be addressed analytically through a least squares problem to compensate the superdirective excitations as shown in (2) where $[E]$ is a column vector of the desired pattern or array factor, $[EEP]$ is a matrix of the embedded element patterns, and $[a]$ is the array excitation. M is the number of observation points and N is the number of elements.

$$[E]_{M\times 1} = [EEP]_{M\times N}[a]_{N\times 1} \rightarrow [a] = ([EEP]^*[EEP])^{-1}[EEP]^*[E] \qquad (2)$$

The same patterns in Fig. 5(a) are plotted in Fig. 6 with the superdirective Cases 1, 2, and 3 excitations compensated using (2). Now the patterns of the superdirective excitations better resemble the desired array factor in Fig. 5(b). The necessity of compensating the superdirective excitations due to their sensitivities further demonstrates the difficulty of using such array weights. More work is needed to investigate the amount and accuracy of the EEP needed to yield good superdirective patterns. For the HF band, due to the difficulty of measuring the EEPs in a deployed array, especially one that can be rapidly stowed and deployed, the superdirective excitations become quite impractical despite their benefits.

Radiation efficiencies are plotted in Fig. 7. Both the uniform amplitude and Hansen-Woodyard excitations are near 100%. However, the three superdirective excitations exhibit the well-known issues with low radiation efficiencies [2], most notable in Case 3.

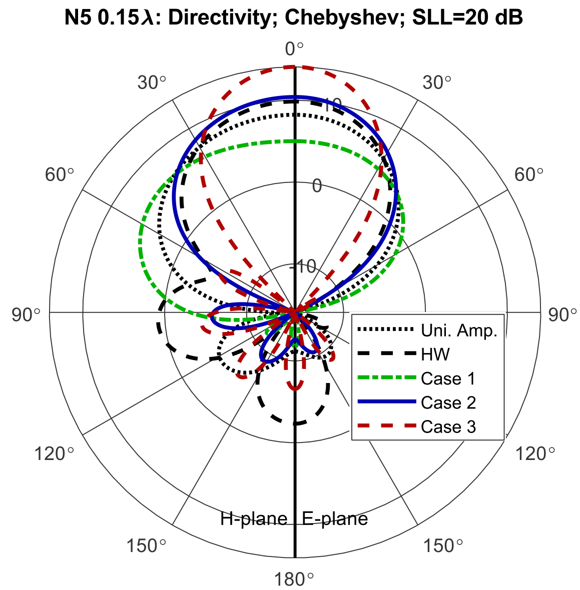


Figure 6. 5 element half-wave dipole array simulated in Altair Feko. 0.15λ spacing. Excitations of Cases 1, 2, and 3 are compensated with the embedded element patterns.

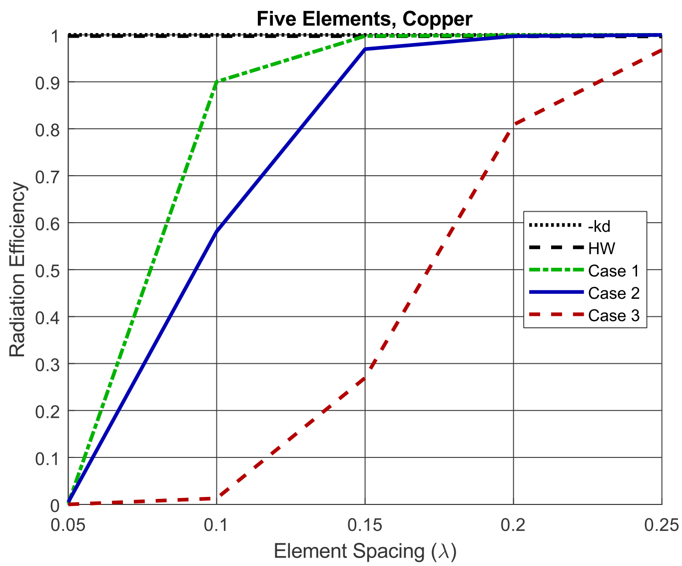


Figure 7. Radiation efficiency of array excitations with 5 element copper half-wave dipoles.

7. Superdirectivity in Arrays of Electrically Small Elements

The size of resonant monopoles in the HF band is physically large, on the order of 2.5m-25m. To this end, electrically small elements are of interest to meet practical size constraints. Two five element linear arrays made from electrically small elements are investigated numerically. The first is an array of short monopoles of height 0.5 m. The second is an array of 6 arm, 2.5 turn spherical folded helices similar to those described by [11]. Both elements have the same radial extent and are operated at 20 MHz, corresponding to an electrical size of ka=0.217. When considered as a single isolated element, both the short monopole and the folded helix have similar far field patterns, but they distribute stored energy differently in their near fields, and the folded helix is investigated as a means of reducing the effects of coupling between array elements.

Each array is considered in the three cases described by [9], [10], the Hansen-Woodyard excitation [5], and uniform excitation. Both arrays have significant mutual coupling between elements, which affects both the element match as well as introducing pattern asymmetry and variation between elements. To achieve the desired pattern for the superdirective excitations, the compensation in (2) is applied based on the EEPs simulated in Altair Feko.

A comparison of the radiation efficiencies resulting from exciting the two arrays in the superdirective and uniform cases is shown in Fig. 8. The folded helix has a larger radiation resistance than the short monopole and can achieve high efficiency as an isolated element even at small electrical sizes [11]. This high efficiency is maintained under larger element spacing but ultimately degrades below 0.3λ spacing and is generally lower for more directive excitation schemes. Due to both the short monopole's poor radiation resistance and the high coupling between elements, the monopole array has poor efficiency across the element spacings considered.

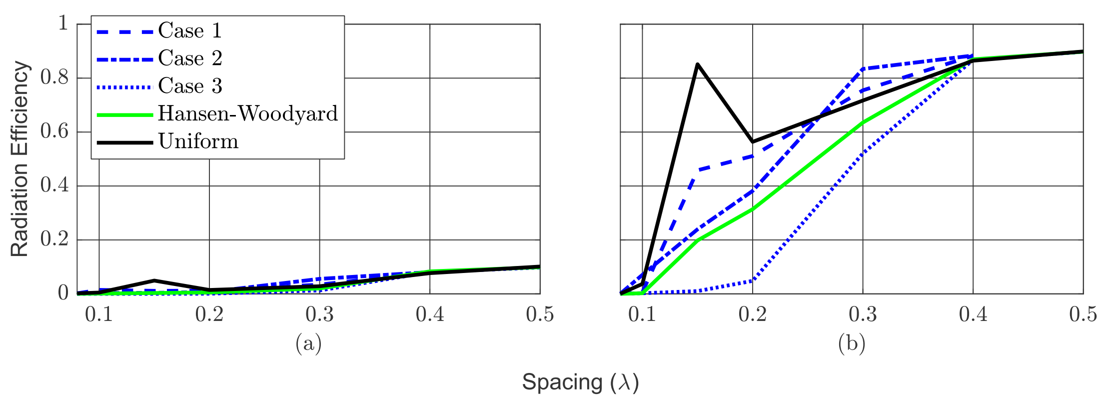


Figure 8. Radiation efficiency of (a) the short monopole and (b) the folded helix arrays.

The directivity achieved by the arrays is shown in Fig. 9. The directivities are similar for both arrays and slightly lower than those for ideal elements, suggesting that the excitation compensation is effective in achieving patterns close to those achieved by coupling-free arrays.

Figs. 10 and 11 compare the effects of excitation amplitude and phase error for the two arrays with 0.15λ spacing. Errors are generated in the same way as described in Section 5 of uniformly distributed errors of ±5% of the maximum amplitude and ±5º. The folded helix array displays substantially less pattern variation than the monopole array and retains stable patterns into some of the superdirective cases. This decreased susceptibility to

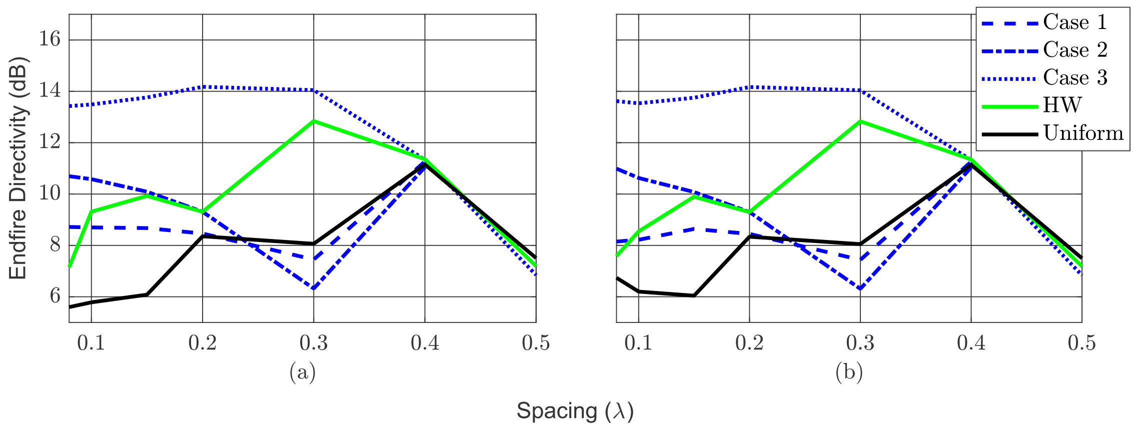


Figure 9. Endfire directivity realized by (a) the short monopole and (b) the folded helix arrays.

excitation errors suggests that the folded helix is successful in reducing the effects of mutual coupling between array elements. In general, however, the superdirective cases remain sensitive to excitation errors in both arrays.

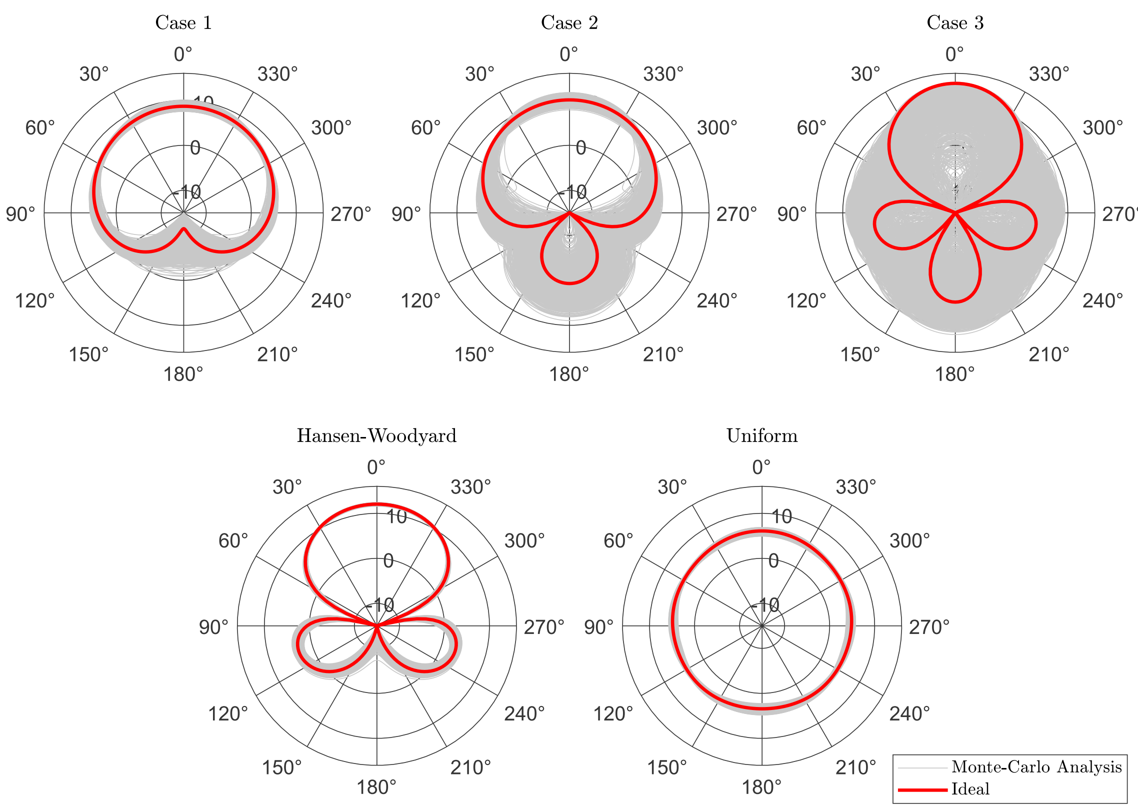


Figure 10. Monte-Carlo analysis of the short monopole array pattern's sensitivity to errors in the excitation amplitude and phase. The array has 0.15λ spacing. For each scheme, the directivity for ideal excitation is compared to that with amplitude and phase errors.

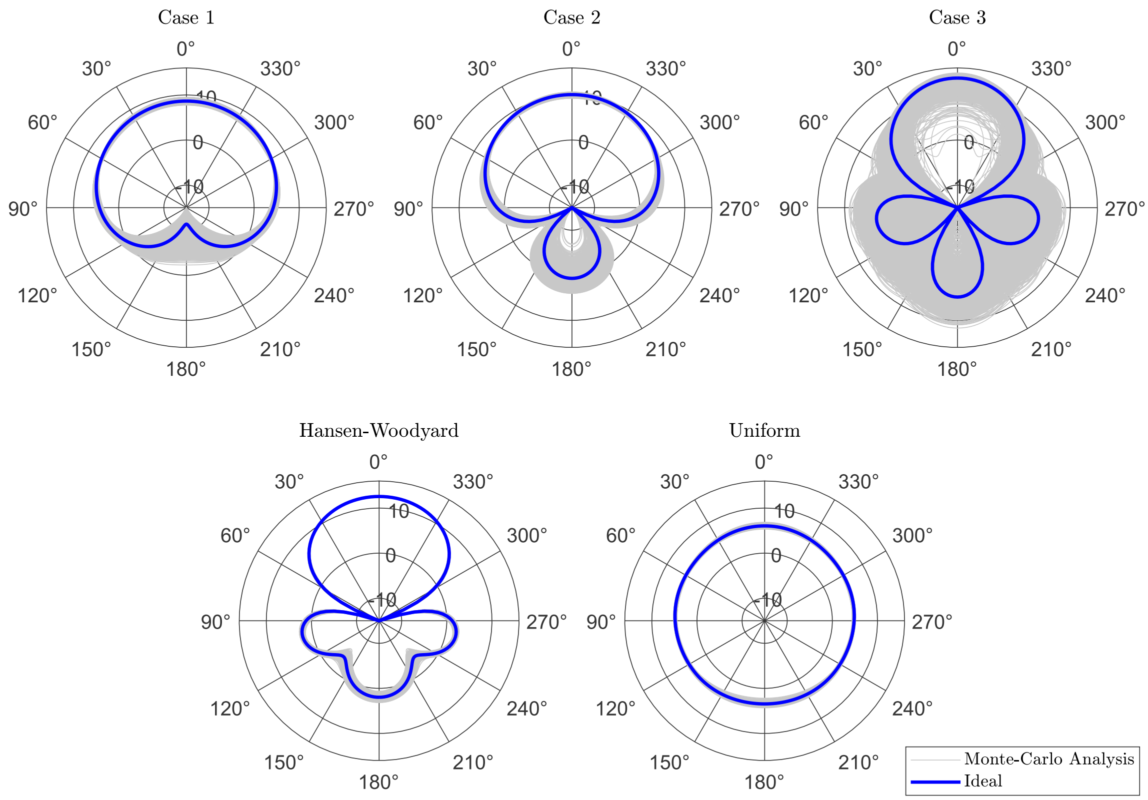


Figure 11. Monte-Carlo analysis of the folded helix array pattern's sensitivity to errors in the excitation amplitude and phase. The array has 0.15λ spacing. For each scheme, the directivity for ideal excitation is compared to that with amplitude and phase errors.

8. Conclusion

Superdirectivity is investigated from a primarily theoretical standpoint for use in the HF band on receive due to the dominance of external noise. Application to simulated half-wave dipole array as well as arrays of electrically small monopoles and folded helix antennas are also included. While attractive in the ideal case with no errors and isotropic elements, the sensitivity of superdirective excitations with implications on both generating the correct amplitudes and phases as well as compensating for realistic EEPs drastically reduces the practicality of superdirective excitations in the HF band. This is especially true as the latter is much harder to accurately obtain. Conversely, the Hansen-Woodyard excitation remains efficient, robust, and largely agnostic to specific EEPs, especially as element number increases. Of course, radiation efficiency must be considered carefully in the transmit mode.

Acknowledgement

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